Final Exam: May 19 Name: _____

You will have 3 hours for this exam, although you should not need that much. This exam is closed-book and closed-note. Please take some time to check your work. If you need extra space, write on the back. There are a total of 70 points on this exam.

1. (12 points) Consider the following Haskell function:

```
f(0) = 1
f(n) = f(n-1) - n + f(n-1) + 2
```

(a) Complete the following table of values of `f( )`:

`n`	0	1	2	3	4	5	6	...	10
`f(n)`								...	

(b) Show the recursion tree (*i.e.*, draw a tree with one node for each function invocation, where the children of a node are the function calls that it makes) for evaluating `f(3)`:

(c) Write a Java version of `f( )` which will produce the same results *and have the same recursion tree*:

(d) Estimate the big-O running time of your function in part (c)

(e) What is an easy way to make this function much more efficient, and what does the big-O running time become?

2. (6 points) What will be the output of the following C++ program?

```
void printem(int n, int a, int &b)
{
    cout << n << ": " << a << " " << b << endl;
    a = 0;
    b = 0;
}

int main()
{
    int x, y;
    int *p, *q;

    x = 4; y = 2; printem(1, x, y);

    x += 3; y += 3; printem(2, x, y);

    p = new int;
    *p = 6; q = &x; printem(3, *p, *q);

    *q = *p; x += 3; printem(4, *p, *q);

    q = p; x += 3; printem(5, *p, *q);

    y = x; x = *p; printem(6, x, y);
}
```

(Recall that `cout << a << b << endl;` is the C++ equivalent of `System.out.print(a); System.out.print(b); System.out.println();`)

3. (2 points) In the above program, what statement needs to be added (and where) to properly deallocate the dynamically allocated memory?

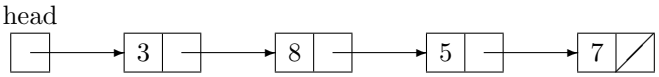
4. (10 points) For these questions, use the following definition for a `Node`:

```
class Node {
    int item;
    Node next;
}
```

(a) Consider the following Java method (in some class that has access to the definition of `Node`):

```
void someFunc(Node head)
{
    Node p = head;
    Node q;
    while (p != null && p.next != null) {
        q = p.next;
        p.item = p.item + q.item;
        p.next = q.next;
        p = p.next;
    }
}
```

Show the result of executing `someFunc(head)` for the following list:



(b) If `someFunc` were written in C++, what additional statement would need to be added (and where) to properly deallocate `Node`s?

(c) Using the above definition of `Node`, write a Java method `countOddPairs(Node head)` which returns a count of the number of pairs of *adjacent* nodes in the list whose sum is odd. For example, calling `countOddPairs(head)` for the above list should return 2, since $3 + 8$ and $8 + 5$ are both odd, while $5 + 7$ is even. Be sure to handle corner cases correctly.

5. (10 points) Here is Java code for one version of the insertion sort algorithm:

```
public void insertionSort(int[ ] a, int n)
// sort a[0 .. n-1] into ascending order
{
    for (int k = 1; k < n; k++) {
        // insert a[k] into the sorted region a[0 .. k-1]
        for (int i = k; i > 0 && a[i - 1] > a[i]; i--) {
            int temp = a[i - 1];
            a[i - 1] = a[i];
            a[i] = temp;
        }
    }
}
```

(a) Trace the operation of insertion sort by showing the contents of `a[ ]` at the end of each pass through the outer loop; use the array `{5, 1, 2, 3, 4}`, where `n = 5`:

k	a[0]	a[1]	a[2]	a[3]	a[4]
initial	5	1	2	3	4
4					
3					
2					
1					

(b) How many item comparisons (`a[i - 1] > a[i]`) are performed in part (a)?

(c) How many swaps are performed in part (a)?

(d) What would be the numbers of comparisons and swaps for the array `{50, 1, 2, ..., 49}`, where `n = 50`?

6. (10 points) Here is Java code for the selection sort algorithm:

```
public void selectionSort(int[ ] a, int n)
// sort a[0 .. n-1] into ascending order
{
    for (int k = n - 1; k > 0; k--) {
        // find index of largest item in a[0 .. k]
        int largest = 0;
        for (int i = 1; i <= k; i++) {
            if (a[i] > a[largest]) largest = i;
        }

        // move largest item to position k
        int temp = a[largest];
        a[largest] = a[k];
        a[k] = temp;
    }
}
```

(a) Trace the operation of selection sort by showing the contents of `a[ ]` at the end of each pass through the outer loop; use the array `{5, 1, 2, 3, 4}`, where `n = 5`:

k	a[0]	a[1]	a[2]	a[3]	a[4]
initial	5	1	2	3	4
4					
3					
2					
1					

(b) How many item comparisons (`a[i] > a[largest]`) are performed in part (a)?

(c) How many swaps are performed in part (a)?

(d) What would be the numbers of comparisons and swaps for the array `{50, 1, 2, ..., 49}`, where `n = 50`?

7. (12 points) Consider the following Haskell code defining a datatype of binary trees, a binary search tree insertion function, and a preorder tree traversal:

```
data Tree = Node(Int, Tree, Tree) | Null

bstInsert(x, Null) = Node(x, Null, Null)
bstInsert(x, Node(y, left, right)) =
    if x == y then Node(y, left, right)
    else if x < y then Node(y, bstInsert(x, left), right)
    else Node(y, left, bstInsert(x, right))

preOrder(Null) = [ ]
preOrder(Node(x, left, right)) = [x] ++ preOrder(left) ++ preOrder(right)
```

(Recall that `++` is the list concatentation operator: `[1, 2] ++ [3, 4]` yields `[1, 2, 3, 4]`.)

(a) Write a Haskell function `find`, with type `(Int, Tree) -> Bool`, such that `find(x, t)` returns `True` if `x` is in the binary search tree `t`, and `False` otherwise:

(b) Complete the following definition of a Haskell (actually HasCl) function `bstInsertAll`, with type `([Int], Tree) -> Tree`, such that `bstInsertAll(xs, t)` is the result of inserting all of the values from the list `xs` into the binary search tree `t`:

```
bstInsertAll([ ], t) = ___________________________

bstInsertAll([x : xs], t) = bstInsert(x, ______________________________________)
```

(c) Give a Haskell definition for an inorder traversal function:

8. (8 points) What is the output when the `main` method of the `Mystery` class is executed?

```
public interface LazyList {
    public int next();
}

public class Ints implements LazyList {
    private int n;

    public Ints(int n)
    {
        this.n = n;
    }

    public int next()
    {
        n++;
        return n;
    }
}

public class Filter implements LazyList {
    private LazyList list;
    private int p;

    public Filter(LazyList list, int p)
    {
        this.list = list;
        this.p = p;
    }

    public int next()
    {
        int n = list.next();
        while (n % p == 0) {
            n = list.next();
        }
        return n;
    }
}
```

```
public class Mystery implements LazyList {
    private LazyList list;

    public Mystery()
    {
        this.list = new Ints(1);
    }

    public int next()
    {
        int p = list.next();
        list = new Filter(list, p);
        return p;
    }

    private static void test(LazyList x)
    {
        for (int i = 0; i < 5; i++) {
            System.out.print(x.next() + " ");
        }
        System.out.println();
    }

    public static void main(String[] args)
    {
        System.out.println("Ints");
        test(new Ints(1));

        System.out.println("Filter");
        test(new Filter(new Ints(1), 2));

        System.out.println("Mystery");
        test(new Mystery());
    }
}
```