

Practice Exam 2

This is a collection of relevant problems I have given on previous exams; it does not correspond exactly to a one-hour test. This exam is closed-book and closed-note. Please take some time to check your work. If you need extra space, write on the back. You must show your work to receive any partial credit.

1. Given the sets

$$A = \{a, i, u\}$$

$$B = \{i, o, u\},$$

list the elements of each of the following sets:

(a) $A \cup B$

(b) $A \cap B$

(c) $A - B$

(d) $(A - B) \cup (B - A)$

(e) $A \times B$

(f) $P(A)$

2. Let A be the set $\{a, e, i, o, u\}$, and consider the relation R on A whose graph is given by the following adjacency matrix:

	a	e	i	o	u
a	0	1	1	0	1
e	0	0	1	0	1
i	0	0	0	0	0
o	0	1	1	0	1
u	0	0	1	0	0

(Recall that the convention is that the cell at row x , column y is 1 if $x R y$.)

- (a) Draw the graph of R :

- (b) Either identify a cycle in the graph of R , or give a topological ordering of the elements of A according to R :

- (c) Which of the following properties apply to R : reflexive, symmetric, transitive, antisymmetric?

3. Given the set $S = \{A, B, C, D, E, F, G\}$, we may represent any subset of S by its characteristic vector, which will have seven bits. For example, the set $\{A, B, D\}$ is represented by the bit vector 1101000. Consider these named subsets of S :

$$I = \{C, E, G\} \quad iii = \{E, G, B\} \quad V = \{G, B, D\}$$

Give the bit vector representation for each of the following:

- (a) $I \cup iii$
- (b) $I \cap iii$
- (c) $I - V$
- (d) $(I - V) \cup (V - I)$
- (e) $(I \cup V) - (I \cap V)$
4. For the same set S , how many elements are in the powerset, $P(S)$? What is the set of bit vectors corresponding to the elements of $P(S)$ (give a simple description)?

5. We observed in class that the logical implication operator, \rightarrow , behaves like a transitive relation. Consider the set \mathcal{E} of all logical expressions. If E and F are elements of \mathcal{E} , then we will define the relation \Rightarrow on \mathcal{E} by saying that $E \Rightarrow F$ exactly when the expression $E \rightarrow F$ is a tautology. For example, $(p + q) \Rightarrow (q + p)$. However, $(p + q) \not\Rightarrow pq$, because when p is true and q is false, the condition $p + q$ is true but the conclusion pq is not.

(a) Does $pq \Rightarrow (p + q)$ hold?

(b) Is \Rightarrow reflexive? Why or why not?

(c) Is \Rightarrow symmetric? Why or why not?

(d) Is \Rightarrow antisymmetric? Why or why not?

(e) Is \Rightarrow transitive? Why or why not?

6. Here is an adjacency list representation of a directed graph:

Node	Successors
A	B
B	(none)
C	A
D	A, B
E	B, D
F	D, G
G	C

(a) Draw the graph.

(b) If the graph is acyclic, give a topological ordering of its nodes; otherwise, identify a cycle.

(c) What is the longest path in the graph that never revisits a node?

(d) Define the distance between two vertices as the length of the shortest path between them, or ∞ if there is no such path. What is the greatest non-infinite distance in this graph?

7. Here is an adjacency matrix representation of an undirected weighted graph (a weight of ∞ means there is no edge between those vertices):

	A	B	C	D	E	F	G
A	0	5	7	4	6	5	2
B	5	0	1	∞	∞	∞	∞
C	7	1	0	8	∞	2	∞
D	4	∞	8	0	1	∞	∞
E	6	∞	∞	1	0	1	∞
F	5	∞	2	∞	1	0	1
G	2	∞	∞	∞	∞	1	0

- (a) Draw the graph.
- (b) Find the (weighted) shortest path from A to E.
- (c) Would the answer to the previous question change if the weight of the edge between B and C were changed to -1 ? Why or why not?
- (d) Find a minimum spanning tree for the graph.